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FORMATION CONDITIONS FOR BUBBLE SUSPENSIONS UPON SHOCK-WAVE LOADING OF LIQUIDS

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The conditions of development of bubble cavitation in liquid media upon shock-wave loading are found. It is s shown that, for the development of an unbounded cavitation, the bubbles should grow to certain critical sizes sufficient for their transition to a nonequilibrium state owing to the elastic energy transferred by a rarefaction wave to a liquid sample (at the stage of unloading). In contrast to low-viscosity liquids, in high-viscosity ones (such as glycerin) these conditions cannot be satisfied for any really attainable parameters of shock waves.

It is known from numerous experimental studies that the stage of fragmentation of a medium is preceded by the growth of cavitational bubbles or pores (in the case of scleronomous materials) upon dynamic failure of low-viscosity liquids [1–3], liquid-disperse and thixotropic media [4, 5], and some plastic metals [6]. Nevertheless, as is shown in [4, 5], cavitational fracture is not always realizable even upon intense pulse loading in liquid media. For example, unlimited bubble growth is not observed during volume pulse expansion of a high-viscosity glycerin sample, and its failure occurs owing to the development of perturbations on the free surfaces. Along with this, when a gel sample that possesses viscoelastic properties fails upon pulse expansion, an unlimited cavitation develops in it and foamy cellular structures form and are then fragmented. In addition, as has already been noted, cavitation is observed in scleronomous materials, for example, in aluminum and copper [7].

In connection with the aforesaid, it seems expedient to determine the conditions under which bubble cavitation develops in condensed media and, thus, to separate a class of media that are capable of cavitating under definite loading regimes. The present study deals with the first stage of investigation of this problem, i.e., the determination of the conditions under which bubble cavitation develops in liquids due to shock-wave loading. The specific features of bubble growth from cavitational nuclei in liquids were considered in a number of publications. For instance, Se Din-Yu [8] studied analytically the growth of a spherical bubble in a viscous incompressible liquid, caused by a short-term pulse of negative ledge-shaped pressure. It is shown that if, for example, a negative pressure equal to $-1.8 \cdot 10^6$ Pa is applied to water during $3 \cdot 10^{-5}$ sec, a cavitational nucleus of radius $R_{00} = 10^{-5}$ cm will grow to the size $R = 10^{-1}$ cm for this time. Person [9] found the conditions under which a single spherical gas-filled bubble grows in a viscous incompressible liquid under the action of an instantaneously applied negative pressure jump in the form of an infinite ledge. The existence of the upper and lower boundaries of negative threshold pressure, which are related to the cases of the infinite and zero viscosity of a liquid, respectively, was established. It was shown that the gas in the bubble exerts a dominant effect on the process considered. Kedrinskii et al. [10] analyzed theoretically the cavitation excitation under the action of a negative pressure pulse of constant amplitude within the framework of the model of an ideal incompressible fluid containing cavitational nuclei with allowance for the results of [8, 9] by referring to the dynamics of a single bubble. It was found that at small amplitudes of unloading one

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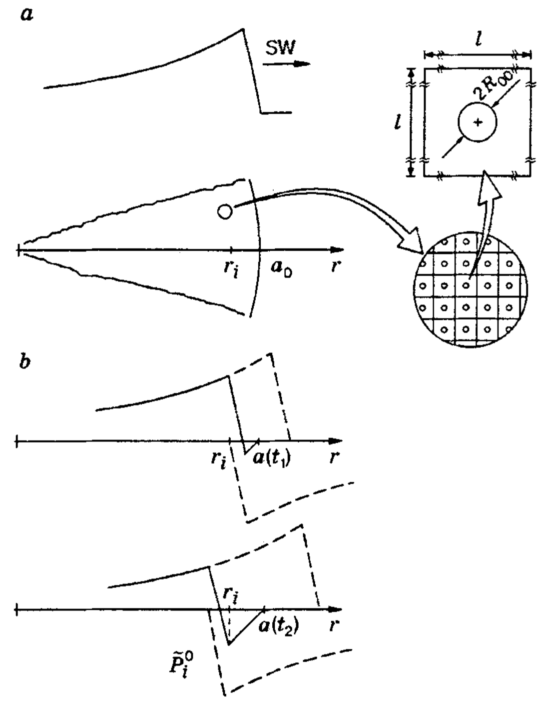

Fig. 1

can observe a strong dependence of the time for which the bubble reaches an appreciable size (of the order 10^{-2} cm) on R_{00}. The interval of R_{00} values that admits this is finite. For large amplitudes, the noticeable sizes are reached by the bubbles almost simultaneously over the entire spectrum of R_{00} values.

The conditions of growth of cavitational nuclei up to noticeable sizes under the action of a ledge of negative pressure instantaneously applied to a medium were considered in [8–10]. However, in the problem of loading a liquid sample by a shock wave (SW), the problem arises of the conditions necessary for the unbounded development of bubble cavitation in a liquid to which a pulse of negative pressure generated after the SW front reaches a free surface is applied. In this case, as is known, the negative-pressure field in a liquid relaxes for a very short time T_0: according to calculated data [11], in the case of water we have $T_0 \sim 10^{-8}$ sec.

1. We now consider the loading of a liquid sample of any viscosity by a short SW so as to find the conditions to which the wave parameters should be subject for the formation of an unbounded cavitation in the sample at the stage of unloading.

We assume that a cylindrical or spherical liquid sample is loaded by a divergent coaxial SW. The characteristic size of the sample is $L = 2a_0$, where a_0 is the initial radius of its cylindrical or spherical free surface (Fig. 1a). Let α_{00} be the initial value of the volumetric concentration of cavitational nuclei in the medium, P_l be the gas pressure in the nuclei, C_0 be the velocity of sound in the liquid matrix, μ be the shear viscosity, ζ_0 be the volumetric (second) viscosity, ρ be the density, γ be the surface tension, and P_∞ be the hydrostatic pressure in the liquid. In the initial state, the liquid contains monodisperse cavitational nuclei for a countable concentration n, i.e.,

$$\alpha_{00} = \left(\frac{3V_0}{4\pi R_{00}^3 n} + 1\right)^{-1}.$$

Here V_0 is the unit volume of the medium. Then, the entire volume of the sample can be divided into n cells of characteristic size l, so that there is one nucleus at the center of each cell. Since in real conditions we have $R_{00} \leqslant 10^{-3}$ cm and $\alpha_{00} = 10^{-12}$–10^{-6}, the growth of a nucleus in a cell in the stretching-stress field to a certain value of α_0 can be regarded as the extension of a spherical bubble in a boundless liquid, because l is several orders of magnitude greater than R_{00}. For example, we have $lR_{00}^{-1} \sim 10^4$ for $\alpha_{00} = 10^{-12}$.

The mechanism of formation of a negative-pressure field in a medium can be presented as follows. A divergent SW propagates along the coordinate r of the spherical (r, θ, φ) or cylindrical (r, φ, z) coordinate system (depending on the geometry of the process). The origins of these systems are aligned, respectively, with the center or the axis of symmetry of the sample. After the SW front reaches the free surface of the sample $r = a_0$ (Fig. 1a), its radial tension begins [$a(t) > a_0$], and the rarefaction wave (RW) reflected from the free surface converges to the center of the system. The dashed curves in Fig. 1b show the profiles of the divergent SW and of the SW convergent to the center of symmetry. If there were no cavitational nuclei in the liquid, a field of negative pressure $\tilde{P}^0(r)$ would be generated behind the RW front (Fig. 1b) and a portion of SW energy would be converted to the elastic energy of the radially stretched medium. However, since we always have $\alpha_{00} > 0$ in the liquid for $|\tilde{P}^0 + P_1| > P_\infty + 2\gamma/R_{00}$, the elastic energy is converted to the work of expansion of the cavitational nuclei, and the pressure $\tilde{P}^0$ relaxes with a certain time constant T_0.

Following to [12], the pressure $\tilde{P}^0$ in a RW can be presented in the form of a superposition

$$\tilde{P}^0(r_*^-, r) = P^+(r_*^+, r) + P^-(r_*^-, r). \tag{1}$$

Here $P^+(r_*^+, r) = f^+(r_*^+, r)U(r-r_*^+)$ is the pressure in the SW divergent from the axis (or center) of symmetry of the sample; $f^+(r_*^+, r) = P_*^+(r_*^+)$ for $r = r_*^+$, $f^+(r_*^+, r) \leqslant P_*^+(r_*^+)$ for $r < r_*^+$, $U(r-r_*^+) = 1$ for $r \leqslant r_*^+$, and $U(r - r_*^+) = 0$ for $r > r_*^+$ and $P_*^+(r_*^+)$ is the pressure in the SW front. Similarly, we present an imaginary SW that converges to the center (or axis) of symmetry of the sample:

$$P^-(r_*^-, r) = f^-(r_*^-, r)U(r - r_*^-),$$

where $|f^-(r_*^-, r)| = |P_*^-(r_*^-)|$ for $r = r_*^-$, $|f^-(r_*^-, r)| \leqslant |P^-(r_*^-)|$ for $r > r_*^-$; $U(r - r_*^-) = 1$ for $r \geqslant r_*^-$, and $U(r - r_*^-) = 0$ for $r < r_*^-$; $P_*^-(r_*^-)$ is the pressure in the front of the imaginary SW, and r_*^+ and r_*^- are the coordinates of the fronts of the divergent and imaginary SW, respectively.

Since $P^+(r_*^+, r = a_1(t)) \leqslant P_*^+(r_*^+)$, with allowance for the condition on the free surface of the sample

$$P^+(r_*^+, a_1(t)) + P^-(r_*^-, a_1(t)) = 0$$

the quantities P^+ and P^- at the point $r_*^- \leqslant r = \tilde{r} \leqslant r_*^+$ can be expressed in terms of their pressure-drop gradients behind the front:

$$P^+(r_*^+, \tilde{r}) = P^+(r_*^+, a_1(t)) + \int_{a_1(t)}^{\tilde{r}} \nabla P^+(r_*^+, r)\, dr,$$

$$P^-(r_*^-, \tilde{r}) = -P^+(r_*^+, a_1(t)) + \int_{a_1(t)}^{\tilde{r}} \nabla P^-(r_*^-, r)\, dr.$$

Substituting these expressions into (1), we obtain a pressure distribution function along r in the RW

$$\tilde{P}^0(r_*^-, \tilde{r}) = -\int_{a_1(t)}^{\tilde{r}} \Big(\Big|\frac{\partial P^+(r_*^+, r)}{\partial r}\Big| + \Big|\frac{\partial P^-(r_*^-, r)}{\partial r}\Big|\Big) dr, \tag{2}$$

that holds for the cylindrical and spherical symmetries. Therefore, after the front of the imaginary SW arrives at the ith cell, a pressure field

$$\sigma_i(r_i, t) = P_1(R) - P_\infty - \tilde{P}_i(r_i, t) - 2\gamma/R$$

is generated in the vicinity of a cavitational nucleus, where $P_1(R) = \Big(P_\infty + 2\gamma/R_{00}\Big)b^{-3k}$, $b = R/R_{00}$, k is the polytropic exponent of the gas in the bubble, and $\tilde{P}_i(r_i, t) = -\tilde{P}_i^0(r_i)f(t)$ is the negative pressure which relaxes by the law $f(t)$ owing to bubble expansion. The growth of a cavitational bubble from a nucleus under the action of σ_i is described by the Rayleigh–Lamb equation [13]:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{4\mu}{\rho}\frac{\dot{R}}{R} = \frac{\sigma_i}{\rho}, \quad R = R_{00} \quad \text{and} \quad \dot{R} = 0 \quad \text{for} \quad t = 0. \tag{3}$$

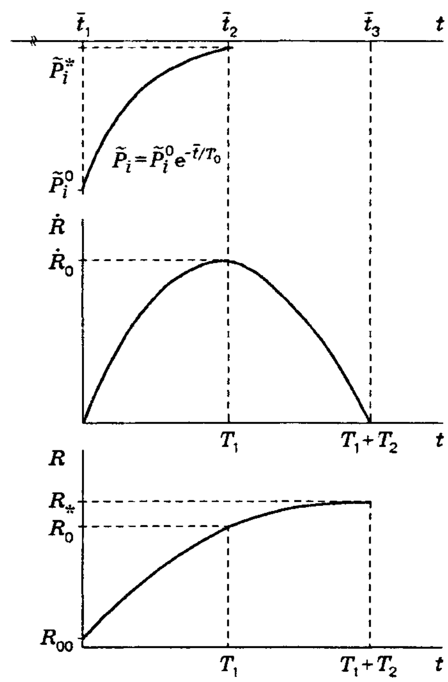

Fig. 2

We distinguish two stages of expansion of a cavitational bubble. On the time scale $\bar{t}$, which is reckoned from the moment of generation of a divergent SW, the first stage begins at $\bar{t} = \bar{t}_1$ [the time when the ith cell reaches the field $\tilde{P}_i^0$ (Fig. 2)] and terminates at $\bar{t} = \bar{t}_2$ which corresponds to relaxation of $\tilde{P}_i$ to $|\tilde{P}_i^*| = P_\infty + 2\gamma/R_0 - P_1$, the value at which the bubble radius increases up to R_0, which corresponds to the maximum expansion rate $\dot{R}_0$. At this moment, the elastic energy of the stretching-stress field is converted to the kinetic energy of the liquid around the bubble in the ith cell and the work to overcome the hydrostatic counterpressure, the surface energy of the bubble, and the thermal energy which is due to the viscosity of the liquid. At the moment $\bar{t}_2$, the second stage of bubble expansion, which occurs in the mode of inertia owing to the stored kinetic energy of a liquid, defined by the bubble-wall velocity $\dot{R}_0$, begins and it terminates at the moment $\bar{t}_3$, when the maximum size $R = R_*$, i.e., $\dot{R}_* = 0$, is attained.

For convenience, we redenote the time scale as follows (Fig. 2): $\bar{t}_1 = t = 0$ is the beginning of the process of extension of a cavitational nucleus, $\bar{t}_2 = t = T_1$ is the onset of the stage of bubble extension by inertia, and $\bar{t}_3 = t = T_1 + T_2$ is the termination of bubble expansion owing to the elastic energy of the stretching-stress field in a liquid cell. Then, the negative pressure in the ith cell can be presented in the form

$$\tilde{P}_i(t) = \tilde{P}_i^0(r_i)\exp(-t/T_0), \qquad \tilde{P}_i^0 < 0, \tag{4}$$

where $\tilde{P}_i^0$ is determined by expression (2). Since T_0 is very small and, hence, the displacements of the fronts of the loading and imaginary SW are insignificant for this time, we assume that the value of $\tilde{P}_i^0(r_i)$ remains constant during relaxation of the pressure $\tilde{P}_i(t)$. Substituting (4) into (3), multiplying the latter by $4\pi R^2\dot{R}$, and integrating over t from 0 to T_1, we obtain

$$-4\pi\tilde{P}_i^0\int_0^{T_1} R^2\dot{R}\exp\Big(-\frac{t}{T_0}\Big)dt = 2\pi\rho R_0^3\dot{R}_0^2 + \frac{4}{3}\pi R_{00}^3\Big[P_\infty(b_0^3-1) - \frac{P_\infty + 2\gamma R_{00}^{-1}}{k-1}(1-b_0^{3-3k})\Big]$$

$$+ 4\pi\gamma R_{00}^2(b_0^2-1) + 16\pi\mu\int_0^{T_1} R\dot{R}^2\,dt. \tag{5}$$

Here $\tilde{P}_i^0 < 0$ and $b_0 = R_0/R_{00}$. With allowance for $\sigma_i = 0$ at $t = T_1$, we have

$$T_1 = T_0 \ln \frac{|\tilde{P}_i^0|}{P_\infty(1 - b_0^{3k}) + 2\gamma(1 - b_0^{-3k})/R_0}.$$

Equality (5) determines the energy balance in the ith cell: the expression on the left side of the equality is the elastic RW energy E_0 spent for bubble growth, the first term on the right side is the gain in the kinetic energy E_1 of a liquid owing to the increase in the bubble-expansion rate to the value of $\dot{R}_0^2$, the second term on the right side is the work of bubble extension in the field of hydrostatic counter pressure E_2, the third term is the gain in the free (surface) energy E_3, and the fourth term is the dissipation of energy E_4 because of the shear viscosity of the liquid.

It has already been mentioned that, according to calculation data [11], the negative pressure in water relaxes owing to the growth of cavitational bubbles for a time less than 10^{-7} sec; under these conditions the bubble fails to reach an appreciable size (the order 10^{-2} cm). According to data of [14], the stretching stress in high-viscosity glycerin relaxes for approximately 10^{-7} sec; here the cavitational nuclei also have no time to reach a noticeable size. Therefore, in (5) the increments of the terms E_2 and E_3 during T_1 can be ignored. With allowance for the fact that $a_1 \simeq a_0$ for $t = T_1$, with E_0 replaced by $\tilde{P}_i^0$ in expressions (2) this relation can be written in the form

$$2\int\limits_{a_0}^{\bar{r}} \Big[\Big|\frac{\partial P^+(r_*^+, r)}{\partial r}\Big| + \Big|\frac{\partial P^-(r_*^-, r)}{\partial r}\Big|\Big] dr \int\limits_0^{T_1} R^2 \dot{R} \exp\Big(-\frac{t}{T_0}\Big) dt \simeq \rho R_0^3 \dot{R}_0^2 + 8\mu \int\limits_0^{T_1} R\dot{R}^2\, dt. \qquad (6)$$

We now estimate the degree of bubble expansion at the stage where $t > T_1$, i.e., in the inertial mode, owing to the stored kinetic energy E_1.

2. We integrate Eq. (3) from R_0 to R_* (the maximum radius attainable by the bubble owing to the stored kinetic energy) on the time interval (T_1, T_2), which corresponds to the inertial stage of bubble expansion, with allowance for $\sigma_i = -P_\infty - 2\gamma R^{-1} + (P_\infty + 2\gamma R_{00}^{-1}) b^{-3k} + \tilde{P}_i^0 \exp(-t/T_0)$, $\dot{R} = \dot{R}_0$ for $t = T_1$ and $\dot{R} = \dot{R}_* = 0$ for $t = T_2$. Resolving the resulting relation with respect to $\rho R_0^3 \dot{R}_0^2$, we substitute it into the right side of (6). As a result, we have a condition under which a cavitational bubble grows from R_{00} to R_* owing to the elastic energy of the stretching-stress field in the ith cell of the sample:

$$3\int\limits_{a_0}^{\bar{r}_i} \Big[\Big|\frac{\partial P^+(r_*^+, r)}{\partial r}\Big| + \Big|\frac{\partial P^-(r_*^-, r)}{\partial r}\Big|\Big] dr \int\limits_0^{T_1} b^2 \dot{b} \exp\Big(-\frac{t}{T_0}\Big) dt$$

$$\geqslant \Big[P_\infty (b_{0*}^3 - 1) - \frac{P_\infty + 2\gamma/R_{00}}{(k+1) b_0^{3k}} (1 - b_{0*}^{-3k-3}) - 3\tilde{P}_i^0 \int\limits_{T_1}^{T_2} b^2 \dot{b} \exp\Big(-\frac{t}{T_0}\Big) dt\Big]$$

$$+ \frac{3\gamma}{R_0} (b_{0*}^2 - 1) + 12\mu \int\limits_0^{T_2} b\dot{b}^2\, dt. \qquad (7)$$

Here $b_{0*} = R_*/R_0$. Let us simplify the left side of this inequality by specifying the profile of a wave that loads the sample as a function $P^+(t) = P_*^+ \exp(-t/\tau)$, where τ is a pressure-drop time constant at a fixed point of the medium. Since, according to numerous experimental data, the development of bubble cavitation in liquid media is excited by shock-loaded wave with an amplitude that does not generally exceed 10^9 Pa, one can consider that the velocity of these SW equals the velocity of sound in the loaded liquid C_0. With allowance for this, after the replacement $t = (r_*^+ - r_i)/C_0$, with the front coordinate r_*^+ fixed, the space pressure distribution behind the SW front can be presented in the form $P_i^+(r_*^+, r_i) = P_*^+(r_*^+) \exp\{-(r_*^+ - r_i)/[\tau(r_i) C_0]\}$; whence

$$\frac{\partial P_i^+}{\partial r} = \frac{P_*^+(r_*^+)}{\tau(r_i) C_0} \exp\Big[-\frac{r_*^+ - r_i}{\tau(r_i) C_0}\Big]. \qquad (8)$$

The pressure on the free surface is always atmospheric; as a result, the negative pressure in the medium behind the RW front can be formed only for $r < a_0$ (see Fig. 1). Therefore, we determine the bubble-growth condition in a cell located at a certain small distance from the free surface δ, so that $\delta/a_0 \ll 1$. The essence of this approach consists of the fact that if the bubble-growth condition holds in a cell with the coordinate $r_i = a_0 - \delta$, it will hold for $r_j < r_i$ as well. With allowance for this, expression (8) for $r_i = a_0 - \delta$ (at the moment of arrival of the SW front at this point, i.e., for $r_*^+ = a_0 + \delta$) can be rewritten as follows:

$$\frac{\partial P_i^+}{\partial r} = \frac{P_*^+(r_*^+)}{\tau(r_i)C_0} \exp\Big[-\frac{2\delta}{\tau(r_i)C_0}\Big]. \tag{9}$$

In the process of convergence of the imaginary SW to the center of symmetry of the sample, the surface of its front decreases; therefore, it is clear that the amplitude at least does not decrease. Therefore, we have $|\partial P^-/\partial r| \geqslant |\partial P^+/\partial r|$ at an equal distance from the sources of waves. Consequently, one can write an inequality that takes the form

$$\int\limits_{a_0}^{a_0-\delta} \Big[\Big|\frac{\partial P^+(r_*^+, r)}{\partial r}\Big| + \Big|\frac{\partial P^-(r_*^-, r)}{\partial r}\Big|\Big] dr \geqslant 2P_*^+(r_*^+)\Big\{1 - \exp\Big[-\frac{\delta}{C_0\tau(r_i)}\Big]\Big\} \tag{10}$$

with account for (8) and (9) and the approximation $\tau(a_0 + \delta) \approx \tau(a_0 - \delta)$ and after integration over r from a_0 to $a_0 - \delta$. Using (10), the bubble-growth condition (7) can be transformed to the inequality

$$3P_*^+(r_*^+)\Big\{1 - \exp\Big[\frac{-\delta}{C_0\tau(a_0 - \delta)}\Big]\Big\} \int\limits_0^{T_1} b^2\dot{b} \exp\Big(-\frac{t}{T_0}\Big) dt \geqslant P_\infty(b_{0*}^3 - 1) - \frac{P_\infty + 2\gamma R_{00}^{-1}}{(k+1)b_0^{3k}} (1 - b_{0*}^{-3k-3})$$

$$- 3\tilde{P}_i^0 \int\limits_{T_1}^{T_2} b^2\dot{b} \exp\Big(-\frac{t}{T_0}\Big) dt + \frac{3\gamma}{R_0}(b_{0*}^2 - 1) + 12\mu \int\limits_0^{T_2} b\dot{b}^2\, dt. \tag{11}$$

Thus, the fulfillment of condition (11) ensures bubble growth in the volume $0 < r \leqslant a_0 - \delta$ of a liquid sample of any shear viscosity. The current values of b and $\dot{b}$ in (11) can be calculated numerically, by solving Eq. (3) on the time interval $(0, T_1)$. The negative-pressure relaxation time $T_0 = T_0(K_0, K_\infty, K_a(\alpha_0), \zeta_0, \zeta_1(\alpha_0))$ is determined from relation (5.5) of [15], which was derived in terms of the macrorheological model of cavitating liquids. Here K_∞ and K_0 are the dynamic and static moduli of volumetric elasticity of a pure liquid and $K_a(\alpha_0)$ is the modulus of volumetric elasticity of a bubble suspension, which is calculated from the relation derived in [15]. The effective volumetric viscosity of a cavitating liquid $\zeta_1(\alpha_0)$ should be found, because the known formula $\zeta_1 = 4\mu/(3\alpha_0)$ [16] holds only for large α_0. If $\alpha_0 \to 0$, then $\zeta_1 \to \infty$, whereas there should be $\zeta_1 \to \zeta_0$ as $\alpha_0 \to 0$. To eliminate the singularity in this formula as $\alpha_0 \to 0$, we should construct the dependence $\zeta_1(\mu, \alpha_0)$ with allowance for the volumetric elasticity of the liquid component.

3. If a real liquid undergoes volumetric expansion so that the Deborah number $\mathrm{De} = T_0/\Delta\tilde{t} \gg 1$, where $\Delta\tilde{t}$ is the characteristic time of increase of the stretching-stress amplitude, the strain of the medium is first determined only by the volumetric strain of the liquid component. After that, the cavitational nuclei grow and, consequently, the stretching stresses begin to relax, i.e., the increase in the volume of the medium is determined only by bubble growth. With allowance for this, we construct a dependence of ζ_1 on the volumetric-strain rate of a pure liquid $\dot{\varepsilon}_{V^0}$ and that of the medium $\dot{\varepsilon}_{VB}$ owing to bubble growth. The energy-dissipation rate in the volume V_0 of a homogeneous liquid D_0 and in a monodisperse bubble suspension that expands only owing to bubble growth D_B is representable in the form [16]

$$D_0 = V_0\zeta_0\dot{\varepsilon}_{V^0}^2, \qquad D_B = 16\pi\mu R\dot{R}^2 N. \tag{12}$$

Here N is the number of bubbles in a medium of volume V^0. Then, in a homogeneous medium rheologically equivalent to the medium considered, the energy-dissipation rate can be presented as follows:

$$D = V^0 \zeta_1 \dot{\varepsilon}_V^2, \qquad D = D_0 + D_B, \qquad \dot{\varepsilon}_V = \dot{\varepsilon}_{V^0} + \dot{\varepsilon}_{VB}. \tag{13}$$

Let us determine the volumetric-strain rate of a pure liquid $\dot{\varepsilon}_{V^0}$ and that of a bubble suspension $\dot{\varepsilon}_{VB}$ in which the change in the volume of the medium is determined only by bubble growth. Since the volume before and after deformation is determined, respectively, by the expressions $V^0 = V_0 + 4\pi R_{00}^3 N/3$ and $V = V_0 + 4\pi R^3 N/3$, with allowance for $n = N/V^0$, the countable bubble concentration, we have

$$\dot{\varepsilon}_{VB} = \frac{d}{dt}\Big(\frac{V - V^0}{V^0}\Big) = \frac{d}{dt}\Big(\alpha_0 - \frac{V^0 - V_0}{V^0}\Big) = \frac{d}{dt}\Big(\frac{4\pi R^3 n}{3} - \alpha_{00}\Big) = 3\alpha_0 \frac{\dot{R}}{R}. \tag{14}$$

We determine the volumetric strain of the liquid component as follows. In the case where De $\gg$ 1, the rheological equation of state of a bubbly suspension [15] becomes a simple relation $\sigma_V = K_a(\alpha_0)\varepsilon_V$, which can be written in the form $\alpha_0 = \alpha_{00}$ for $K_a(\alpha_{00}) \simeq K_\infty$ with allowance for $\sigma_{V^0} = K_\infty \varepsilon_{V^0}$; whence

$$\dot{\varepsilon}_{V^0} = \dot{\sigma}_{V^0} K_\infty^{-1}. \tag{15}$$

To find $\dot{\sigma}_{V^0}$, we use the pressure expression at a fixed point in the medium r_i through which a SW passes: $P^+(r_i, t) = P_*^+(r_i)\exp(-t\tau^{-1}(r_i))$. We consider that the SW front increases for $\Delta t_{\rm f}$, which is of the order of 10^{-7}–10^{-6} sec according to experimental data. Then, at the moment when the imaginary SW arrives at the point r_i, its profile can be presented in the form

$$P^-(r_i, \hat{t}) = \Big\{P_*^-(a_0 - \delta)[U(\hat{t}) - U(\hat{t} - \Delta t_{\rm f})](\hat{t}/\Delta t_{\rm f}) + P_*^-(a_0 - \delta)U(\hat{t} - \Delta t_{\rm f})\Big\}$$
$$\times \exp[-\hat{t}U(\hat{t} - \Delta t_{\rm f})/\tau(a_0 - \delta)]. \tag{16}$$

Here $P_*^- < 0$; $\hat{t}$ is the time from the moment of arrival of the imaginary SW at the point $r_i = a_0 - \delta$ and $U(\hat{t}) = 0$ and 1, respectively, for $\hat{t} < 0$ and $\hat{t} \geqslant 0$ and $U(\hat{t} - \Delta t_{\rm f}) = 0$ and 1 for of $\hat{t} \leqslant \Delta t_{\rm f}$ and $\hat{t} > \Delta t_{\rm f}$, respectively. The character of pressure increase in the front of the divergent i does not exert an effect on the formation of a negative pressure in the $P^+(r_i, \hat{t})$th cell; therefore, for $r_i = a_0 - \delta$, one can write

$$P^+(r_i, \hat{t}) = P_*^+(a_0 - \delta)\exp[-(2\delta C_0^{-1} + \hat{t})/\tau(r_i)]. \tag{17}$$

Since $\delta/a_0 \ll 1$, we have $\tau(a_0 + \delta) \simeq \tau(a_0 - \delta) \simeq \tau$. With allowance for (16) and (17), the determining negative pressure in the ith cell takes the form

$$\tilde{P}(r_i, \hat{t}) = \begin{cases} P_*^+(a_0 - \delta)\exp\Big(\dfrac{-2\delta/C_0 - \hat{t}}{\tau}\Big) - P_*^-(a_0 + \delta)\dfrac{\hat{t}}{\Delta t_{\rm f}}, & 0 \leqslant \hat{t} \leqslant \Delta t_{\rm f}, \\ P_*^+(a_0 - \delta)\exp\Big(\dfrac{-2\delta/C_0 - \hat{t}}{\tau}\Big) - P_*^-(a_0 + \delta)\exp\Big(-\dfrac{\hat{t}}{\tau}\Big), & \hat{t} > \Delta t_{\rm f}. \end{cases} \tag{18}$$

Then, substituting $\dot{\sigma}_{V^0}(r_i, \hat{t}) = -d\tilde{P}(r_i, \hat{t})/d\hat{t}$, where $\tilde{P}(r_i, \hat{t})$, is determined from (18), into (15), we have

$$\dot{\varepsilon}_{V^0}(r_i, \hat{t}) = \begin{cases} \dfrac{P_*^+(a_0 - \delta)}{\tau K_\infty}\exp\Big(-\dfrac{2\delta C_0^{-1} + \hat{t}}{\tau}\Big) + \dfrac{P_*^-(a_0 + \delta)}{K_\infty \Delta t_{\rm f}}, & 0 \leqslant \hat{t} \leqslant \Delta t_{\rm f}, \\ \Big[P_*^+(a_0 - \delta)\exp\Big(-\dfrac{2\delta}{C_0\tau}\Big) - P_*^-(a_0 - \delta)\Big]\dfrac{\exp(-\hat{t}/\tau)}{K_\infty \tau}, & \hat{t} > \Delta t_{\rm f}. \end{cases} \tag{19}$$

Finally, substituting expressions (12), (14), and (19) into (13), with allowance for $|P_*^+(a_0+\delta)/P_*^-(a_0-\delta)| \simeq 1$ and $\Delta t_{\rm f}/\tau \ll 1$ and, therefore, for $\hat{t} \leqslant \Delta t_{\rm f}$ $\exp(\hat{t}\tau^{-1}) \simeq 1$, we obtain

$$\zeta_1 = \begin{cases} \dfrac{\zeta_0}{V^0(1 + \Theta)^2/V_0} + \dfrac{4\alpha_0 \mu}{3(\alpha_0 + \Theta^{-1})^2}, & 0 \leqslant \hat{t} \leqslant \Delta t_{\rm f}, \\ \dfrac{\zeta_0}{V^0(1 + \Psi)^2/V_0} + \dfrac{4\alpha_0 \mu}{3(\alpha_0 + \Psi^{-1})^2}, & t > \Delta t_{\rm f}, \end{cases} \tag{20}$$

where $\Theta = \dfrac{3\alpha_0 K_\infty \Delta t_{\rm f} \dot{R}/R}{P_*^+(a_0 - \delta)}$ and $\Psi = \dfrac{3\alpha_0 K_\infty \tau \dot{R} R^{-1}\exp(\hat{t}\tau^{-1})}{P_*^+(a_0 - \delta)[\exp(-2\delta/(\tau C_0)) - 1]}$. As $\alpha_0 \to 0$, according to (20) we have $\zeta_1 \to \zeta_0$, whereas, for $\Theta \gg 1$ at the stage $0 \leqslant \hat{t} \leqslant \Delta t_{\rm f}$ or $\Psi \gg 1$ at the stage $\hat{t} > \Delta t_{\rm f}$, we have

$\zeta_1 \simeq 4\mu/(3\alpha_0)$, which corresponds to the known formula [16] that determines the volumetric viscosity of a concentrated bubble suspension. The resulting expression (20) allows us to determine ζ_1 at the initial stage of cavitational-nucleus growth in the mode of pulse expansion of a cavitating liquid and, thus, completely determines the macrorheological equation of state [15].

4. At the stage where $t > T_1$, the counteraction of the hydrostatic pressure P_∞ to bubble expansion is partially compensated, according to (11), by the "residual" negative pressure and the gas pressure in the bubble. Therefore, to estimate from above the elastic-energy store (transferred by the RW to the liquid) necessary for bubble growth to the size R_*, the effect of $\tilde{P}_i^0 \exp(-t/T_0)$ and $P_1(R)$ at $t > T_1$ can be ignored, and Eq. (11) can be reduced to the form

$$3P_*^+(a_0+\delta)\Big\{1-\exp\Big[\frac{-\delta}{C_0\tau(a_0-\delta)}\Big]\Big\}\int_0^{T_1} b^2\dot{b}\exp\Big(-\frac{t}{T_0}\Big)dt \geqslant P_\infty(b_{0*}^3-1)+\frac{3\gamma}{R_0}(b_{0*}^2-1)+12\mu\int_0^{T_1} b\dot{b}^2\,dt. \qquad (21)$$

In the case of low-viscosity liquids ($\mu \approx 0$), from (21) we obtain the following condition for bubble growth to the size R_*:

$$3P_*^+(a_0+\delta)\Big\{1-\exp\Big[\frac{-\delta}{C_0\tau(a_0-\delta)}\Big]\Big\}\int_0^{T_1} b^2\dot{b}\exp\Big(-\frac{t}{T_0}\Big)dt \geqslant P_\infty(b_{0*}^3-1)+\frac{3\gamma}{R_0}(b_{0*}^2-1). \qquad (22)$$

From (18), with allowance for $P_*^+(a_0-\delta) \simeq P_*^-(a_0+\delta)$ we have $\tilde{P}(r_i,T_1) \simeq P_*^+(r_i)[1-\exp(-2\delta/(C_0\tau))]\exp(-T_1/\tau)$; since $\sigma_i(T_1)=0$, we have $(P_\infty+2\gamma/R_{00})b_0^{-3k} - P_\infty - 2\gamma/R_0 + P_*^+(r_i)[1-\exp(-2\delta\tau^{-1}/C_0)]\exp(-T_1/\tau)=0$. Therefore, since we usually have $\tau > 10^{-6}$ sec and T_1 is of the same order as T_0, one can calculate $\exp(-T_1/\tau) \simeq 1$ and R_0 from the expression

$$(P_\infty+2\gamma/R_{00})b_0^{-3k} - P_\infty - 2\gamma/R_0 + P_*^+(r_i)=0, \qquad (23)$$

by setting the values of R_{00} and $P_*^+(r_i)$ and the coordinate $r_i = a_0-\delta$.

In the case of high-viscosity liquids, ignoring the effect of the surface tension and hydrostatic pressure, one can write Eq. (3) for $t \geqslant T_1$ in the form $R\ddot{R}+3\dot{R}^2/2+4\mu\dot{R}/(\rho R)=0$. From here, with allowance for $\dot{R}_*=0$ we have the expression $\rho R_0^3\dot{R}_0^2 = 64\mu^2R_0(b_{0*}^{1/2}-1)^2/\rho$; with this expression substituted into (6) and with allowance for (10), we obtain the condition of bubble growth in a high-viscosity liquid:

$$P_*^+(a_0+\delta)\Big\{1-\exp\Big[\frac{-\delta}{C_0\tau(a_0-\delta)}\Big]\Big\}\int_0^{T_1} b^2\dot{b}\exp\Big(-\frac{t}{T_0}\Big)dt \geqslant \frac{16\mu^2}{\rho R_0^2}(b_{0*}^{1/2}-1)^2+2\mu\int_0^{T_1} b\dot{b}^2\,dt. \qquad (24)$$

Therefore, setting the initial physical parameters of a liquid sample and the parameters of a loading SW, one can find ζ_1 by formula (20) and calculate the negative-pressure relaxation time T_0 in the medium to be examined by means of relation (5.5) in [15]. Further, with allowance for T_0 and the value of R_0 from (22) or (24), which was calculated from (23), for given SW parameters, the limiting radius of the growing bubble R_* is determined or, vice versa, the desired parameters of the loading SW are found by means of R_*.

A qualitative analysis of inequalities (21), (22), and (24) shows the following. The greater the SW time constant τ, the greater the wave amplitude for the cavitation-development condition to be fulfilled, which is in agreement with experimental results [12]. When the sample is loaded by a stepwise SW, we have SW$\tau \to \infty$, and the left side of the above inequalities tends to zero. In this case, cavitation that is due to the elastic energy stored in the medium in the pulse mode cannot occur. With decrease in τ, the amplitude P_*^+ necessary for fulfilling the conditions of the inequalities decreases; however, if $\tau \ll a_0C_0^{-1}$, the cavitation development zone in the sample is of a local nature. Therefore, the case where $\tau \simeq a_0C_0^{-1}$ and the appropriate condition (21), (22), or (24) is satisfied is optimal for cavitation.

In low-viscosity liquids, we have $T_0 < 10^{-7}$ sec and, according to the estimate by (3), the nucleus radius increases by less than one order of magnitude for this time even at $|\tilde{P}_i| \approx 10^9$ Pa. At the subsequent stage of inertial bubble expansion, the radius can reach 10^{-2} cm only in the case $|\tilde{P}_i| > 10^9$ Pa. The growth of bubbles to appreciable sizes, which is observed experimentally in low-viscosity liquids for $P_*^+ \ll 10^9$ Pa, and their consequent growth occurs during T_2, which is 3 or 4 orders greater than the value of T_0. The mechanism of formation of unbounded bubble cavitation in low-viscosity liquids consists of the following. It is known that any, including volumetric, deformation of liquids is accompanied by flow of the medium owing not only to bubble expansion, but primarily owing to the conversion of a portion of the SW energy to the kinetic energy of a divergent flow, which is caused by the displacement of the free surface. Obviously, this flow has a velocity gradient in the unloading zone. Therefore, a prolonged (compared to T_0) expansion of the medium, which compensates for the hydrostatic counter pressure, forms during the process of flow, thus reducing the energy consumption for bubble expansion. As a result, at the stage of $t > T_1$ the bubbles expand under the action of an additional negative pressure $-P_i$ applied to the liquid, rather than in the pure inertial mode. As is known from [9, 10, 13], the cavitational nucleus loses equilibrium and begins to grow unboundedly if a constant pressure with an amplitude smaller than a certain critical value for this value of $R_{00} = R_{\rm cr}$ is applied to the medium. The viscosity can influence the bubble state through dissipation of the kinetic energy of the surrounding liquid only during the time variation of its radius. Therefore, if a bubble remains in equilibrium in a low-viscosity liquid at a given pressure, the more so it ceases to grow at this pressure in a high-viscosity liquid.

With allowance for the aforesaid, an intense cavitation that is characterized by values of the bubble radii not smaller than a certain specified R^0 can develop in a sample of low-viscosity liquid upon its shock-wave loading in two cases:

— The SW parameters P_*^+ and τ satisfy the bubble-growth condition (22) at the stage where $0 \leqslant t \leqslant T_2$ to $R_* \geqslant R^0$;

— According to condition (22), the SW parameters P_*^+ and τ correspond, at the stage where $0 \leqslant t \leqslant T_2$, to the moment where the bubbles reach the sizes $R_* \geqslant R_{\rm cr}$, where $R_{\rm cr}$ corresponds, according to [9, 10]), to the unbounded bubble growth in a given negative-pressure field, which is due to a velocity-gradient flow behind the mobile free surface of a liquid.

In the case of a high-viscosity liquid, according to (24) the energy dissipation increases proportionally to μ^2. Therefore, according to (23), even for $b_0 = 100$ and $R_{00} = 10^{-4}$ cm and a very high expansion rate $\dot{R}_0$ (for example, of the order of 10^4 cm/sec) a bubble in glycerin increases no more than fivefold at the stage of inertial extension. Thus, in practice, a noticeable bubble size cannot be reached upon pulse expansion of glycerin owing to loading by a real SW, which was observed in earlier experiments [4, 5]. Slow formation of a bubble suspension does not occur either, since the negative pressure caused by a gradient flow behind the mobile free surface and the bubble sizes R_*, according to [9, 10, 13], do not satisfy the condition of their slow growth.

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